*Patchy habitats on Prince Edward Island, by Joanne Clifford (CC BY 2.0)*

# Habitat loss, extinction debts, and opportunities for restoration

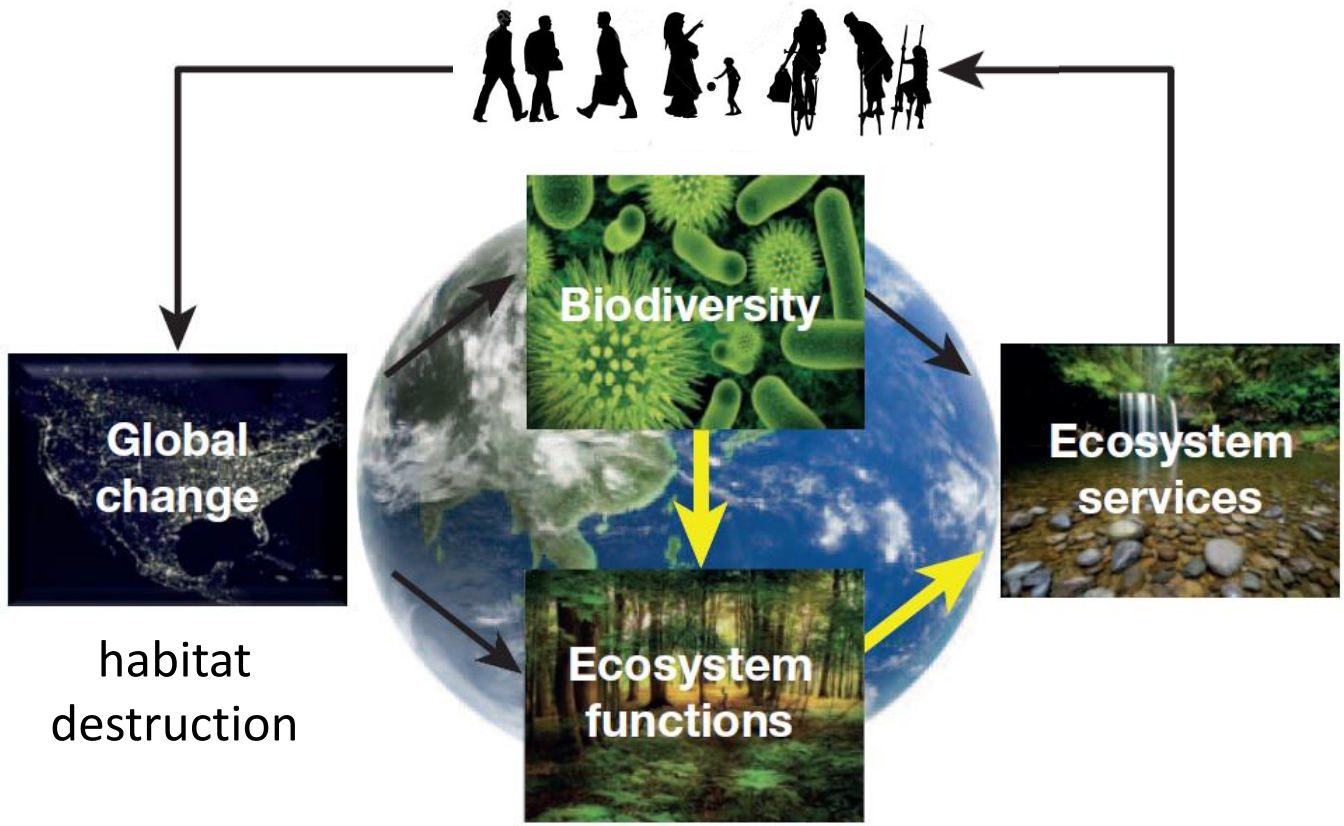
Kate Meyer

Math Climate Seminar

February 26, 2019

nature
International journal of science
REVIEW

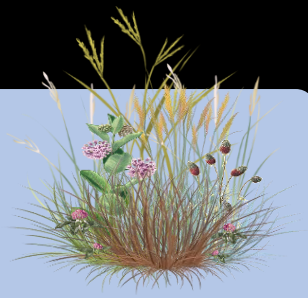
# Biodiversity loss and its impact on humanity




adapted from Cardinale et al. (2012)

# What can math reveal about...

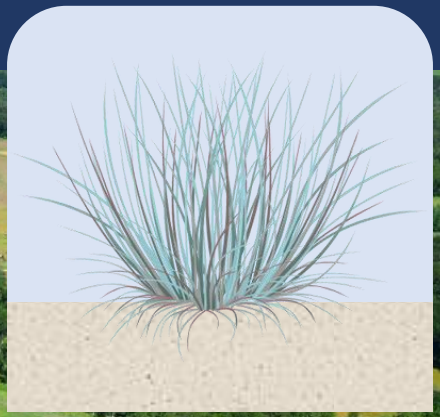
**WHY** biodiversity exists,



**HOW** habitat loss drives extinctions, and

**WHAT** can be done about it?

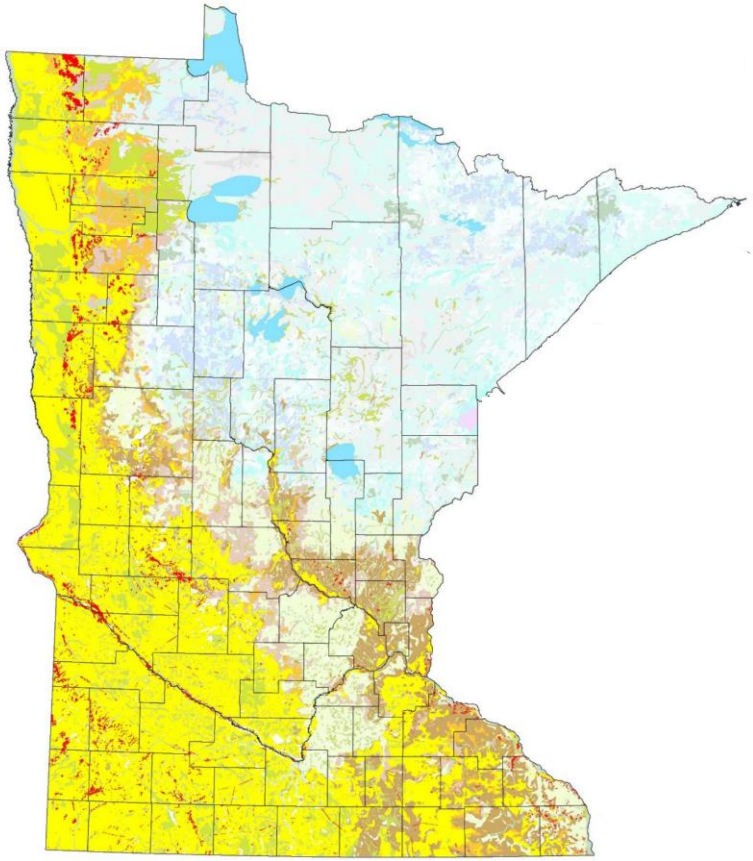
# Why so many species?



# Implicit space

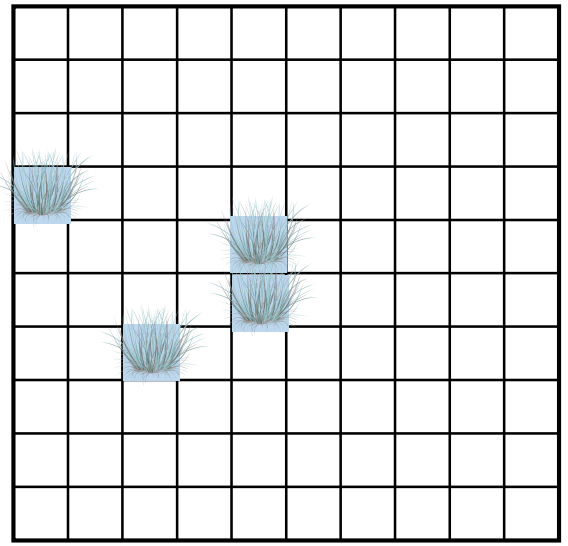
$p$ : proportion of sites occupied

**metapopulation**

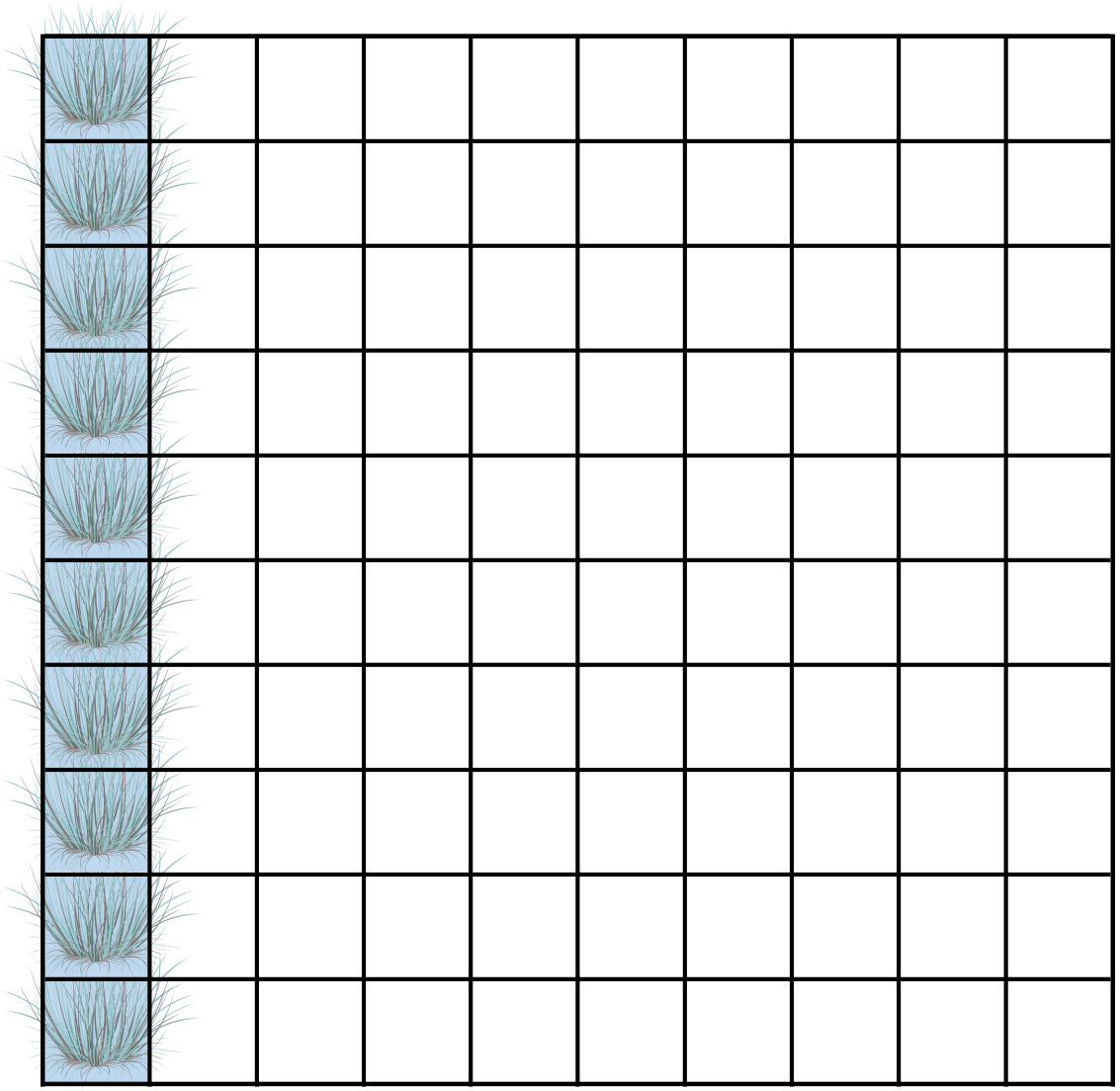


nature.org/media/minnesota/mn-prairie-conservation-plan.pdf

**grid-like habitat**

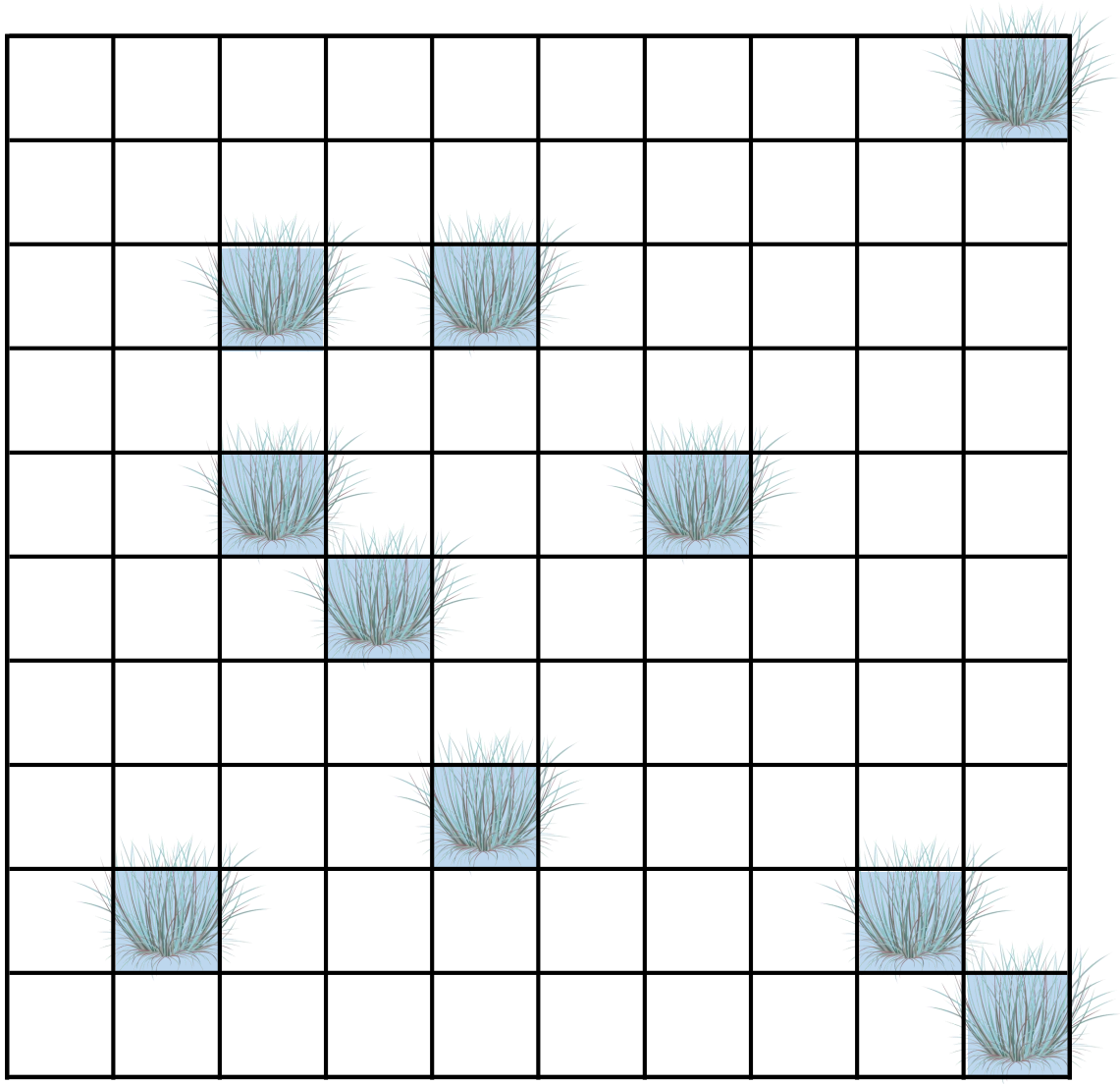


# Implicit space



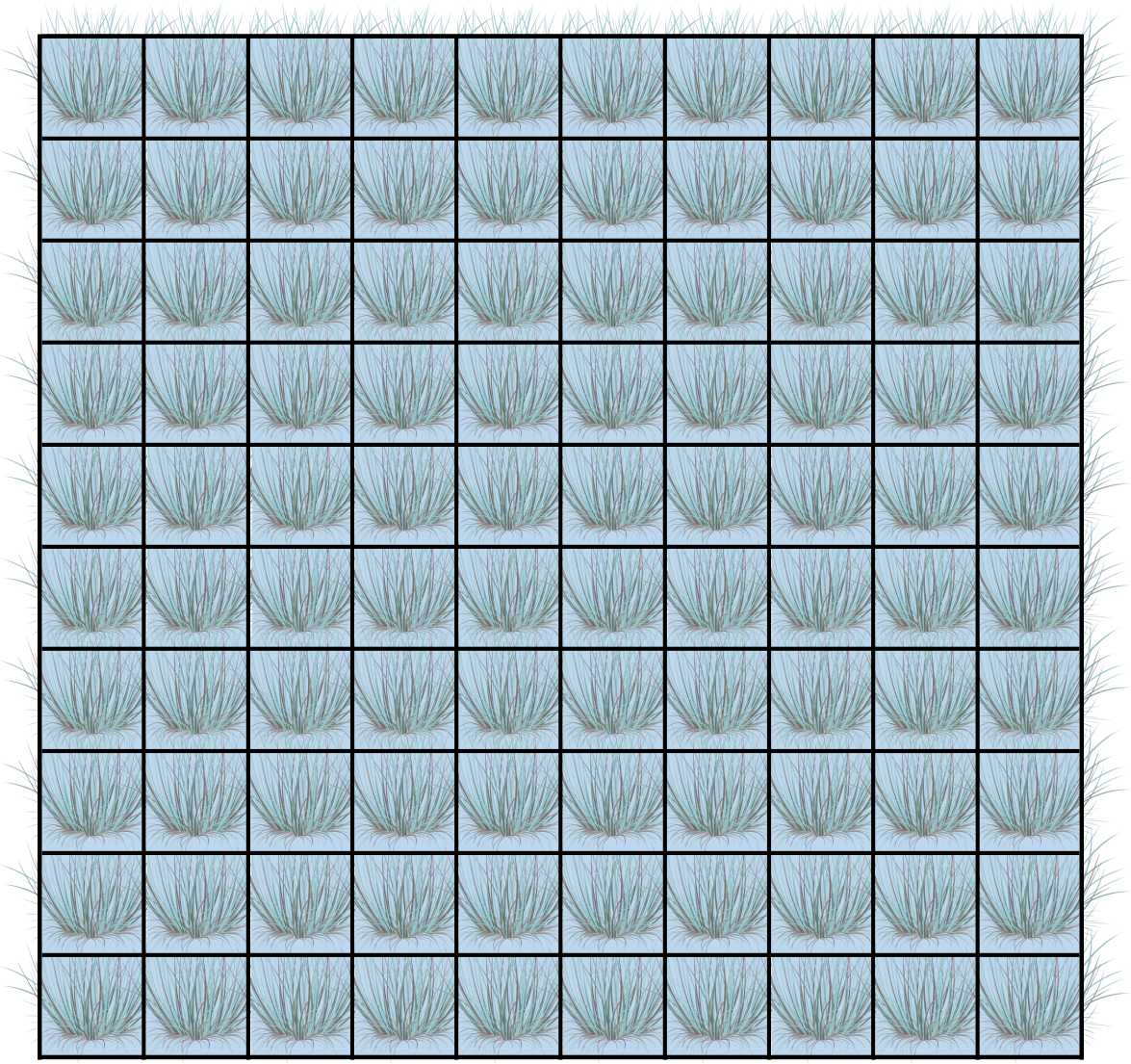
$p = 0.1$

# Implicit space



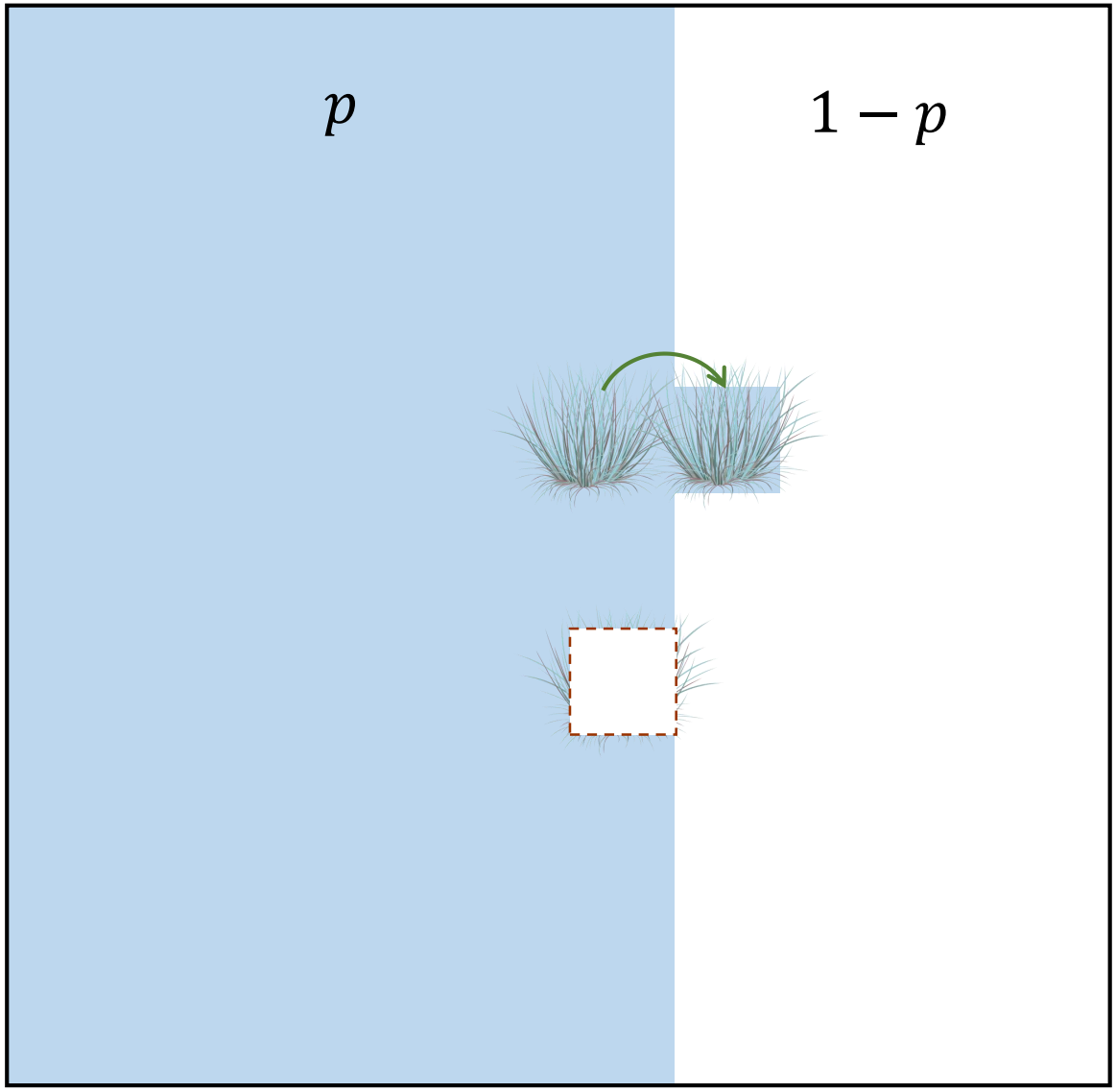
$p$ =0.1

# Implicit space



$p = 1$

# Colonization – mortality dynamics




$$\frac{dp}{dt} = \overbrace{cp(1-p)}^{\text{colonization}} - \underbrace{mp}_{\text{mortality}}$$

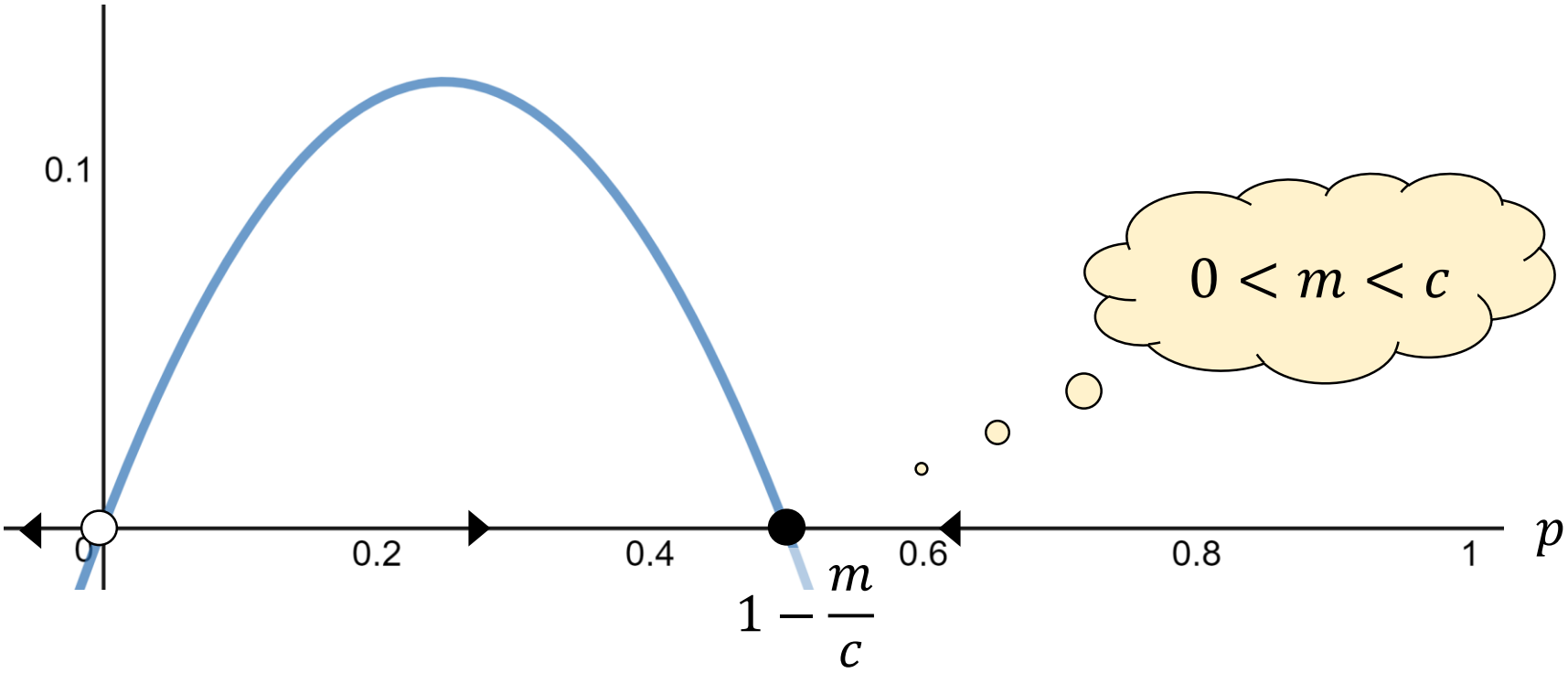
Levins & Culver (1971)

# Colonization – mortality dynamics

$$\frac{dp}{dt} = cp(1-p) - mp$$

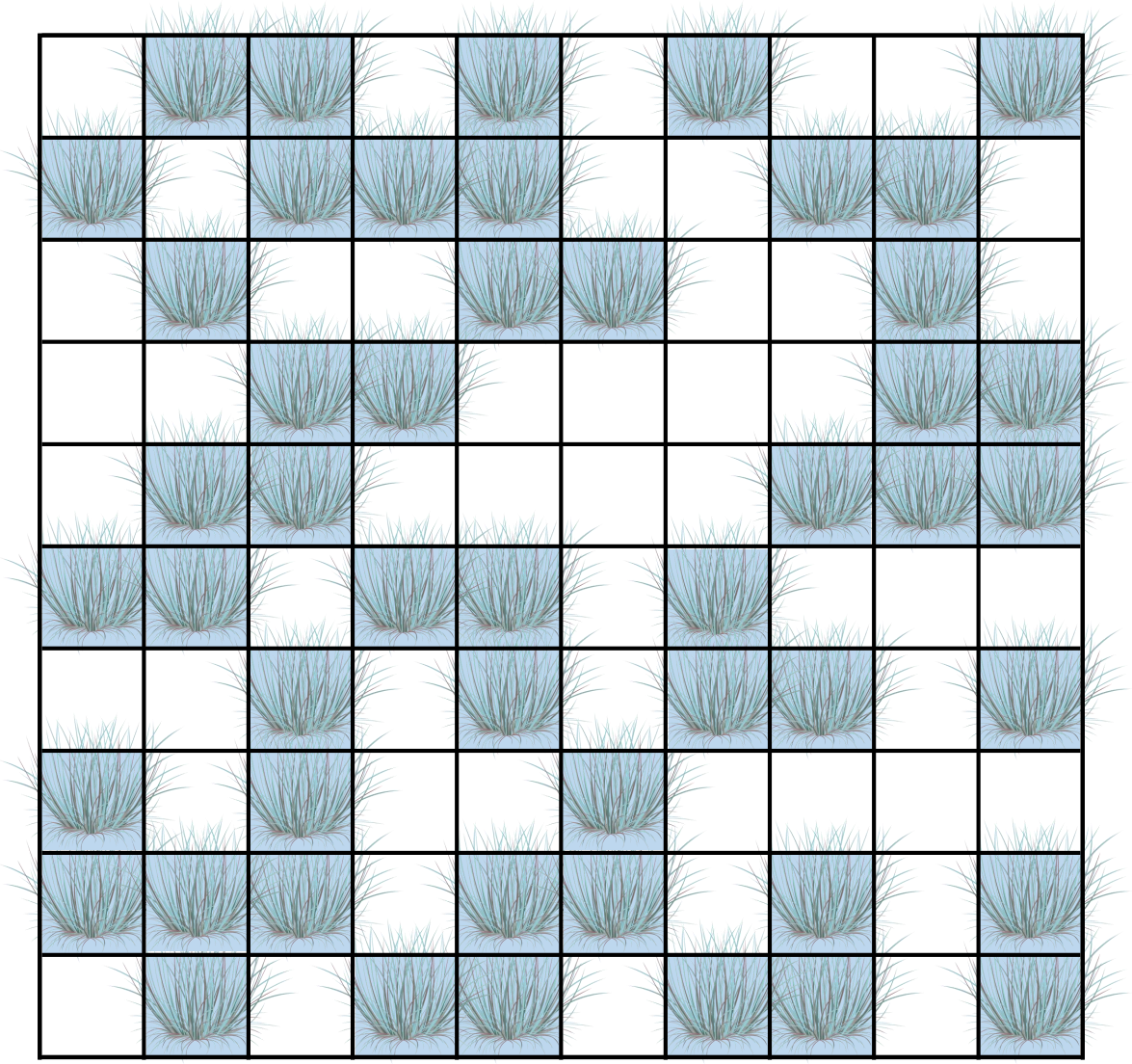
illustration:

$c = 2, \quad m = 1$




**Equilibria:** $\frac{dp}{dt} = 0 \quad \Rightarrow \quad p^* = 0, \;\; p^* = 1 - \frac{m}{c}$

# A superior competitor can leave space for others

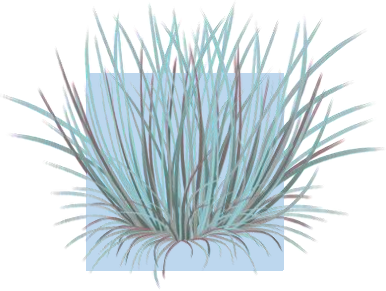


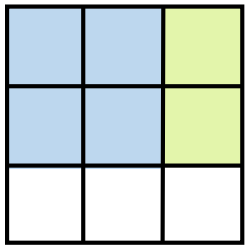
$$p^* = 1 - \frac{m}{c} = 0.5$$

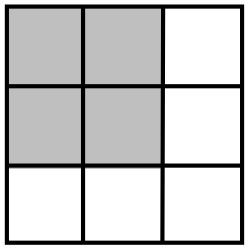
# Two species competitive hierarchy

**“reality”**

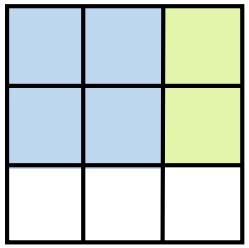
**species’ view**

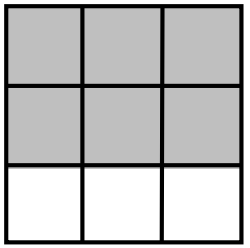
1. 





2. 





$$\frac{dp_1}{dt} = c_1 p_1 (1 - p_1) - m_1 p_1$$

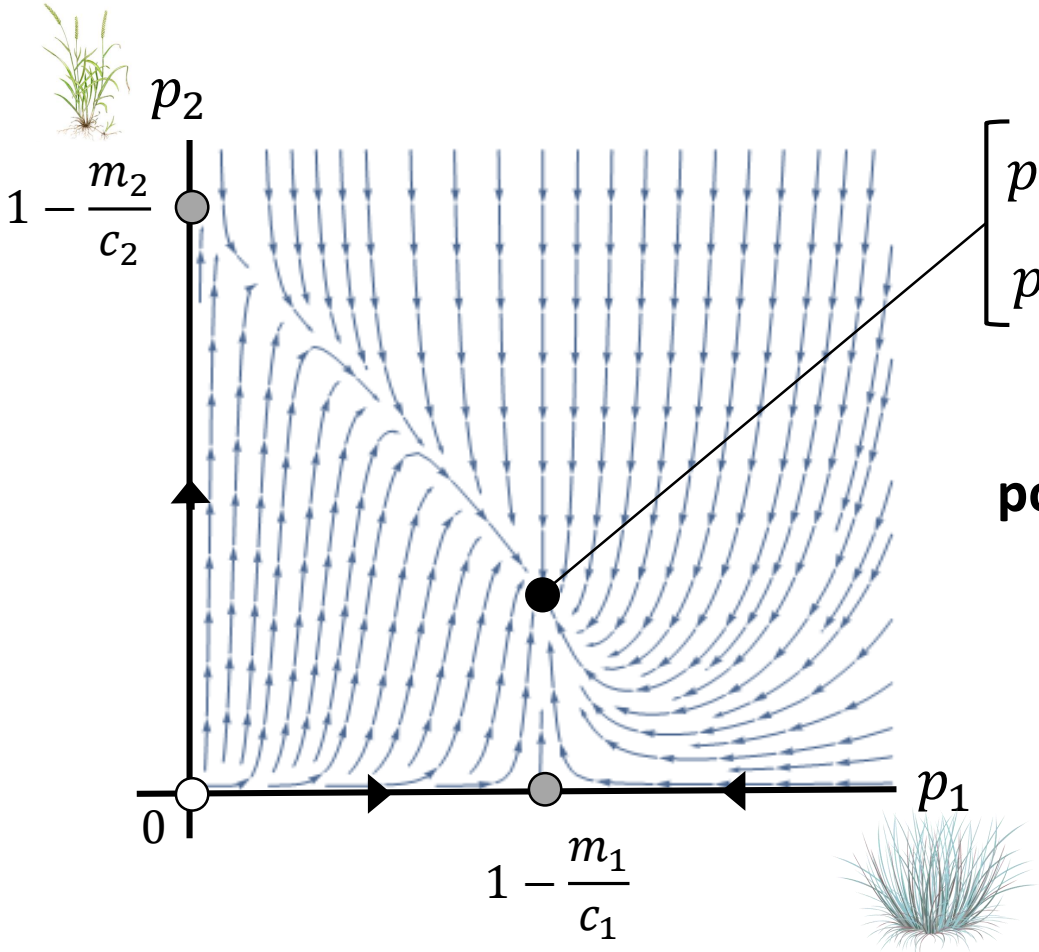
$$\frac{dp_2}{dt} = c_2 p_2 (1 \qquad - p_2) - m_2 p_2$$

# Two species competitive hierarchy

$$p_1' = c_1 p_1 (1 - p_1) - m_1 p_1$$

$$p_2' = c_2 p_2 (1 - p_1 - p_2) - m_2 p_2 - c_1 p_1 p_2$$

illustration:
$c_1 = 2, \quad m_1 = 1$
$c_2 = 10, \quad m_2 = 1$




$$\begin{bmatrix} p_1^* = 1 - \frac{m_1}{c_1} \\ p_2^* = 1 - \frac{m_2}{c_2} - \left(1 + \frac{c_1}{c_2}\right)\left(1 - \frac{m_1}{c_1}\right) \end{bmatrix}$$

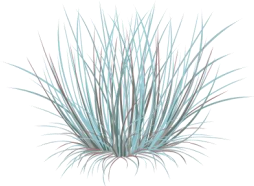
**positive and locally stable when**

$0 < m_1 < c_1,$

$m_1 = m_2 = m,$

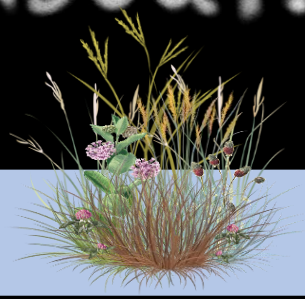
$c_2 \gg c_1$

# Takeaway



Coexistence is possible when species 1 is a superior local competitor and species 2 is a superior colonizer

# What can math reveal about...



**WHY** biodiversity exists?

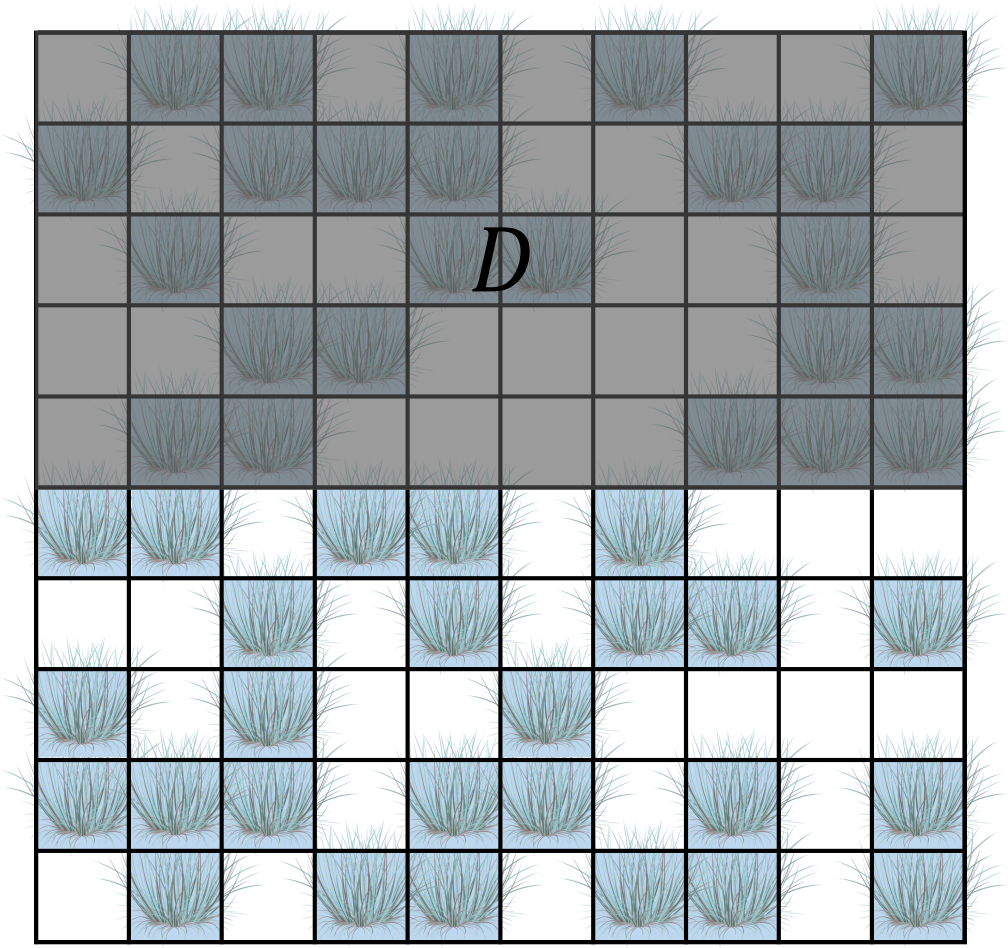
**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

How much is
too much?

# Modeling habitat destruction

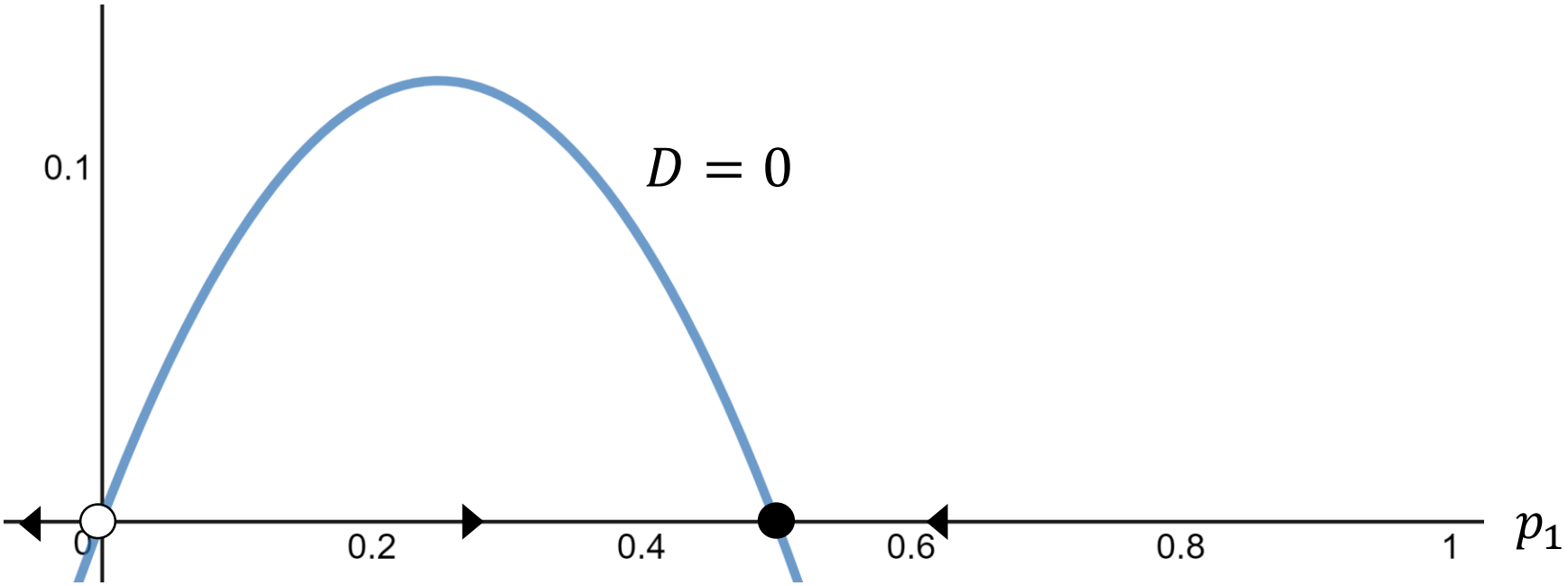
$$p_1' = c_1 p_1 (1 - D - p_1) - m_1 p_1$$



# Habitat destruction: how much is too much?

$$\frac{dp_1}{dt} = c_1 p_1 (1 - D - p_1) - m_1 p_1$$

illustration:
$c_1 = 2, \quad m_1 = 1$




**Equilibria:** $\frac{dp}{dt} = 0 \quad \Rightarrow \quad p^* = 0, \;\; p^* = 1 - \frac{m}{c} - D$

# Habitat destruction: how much is too much?

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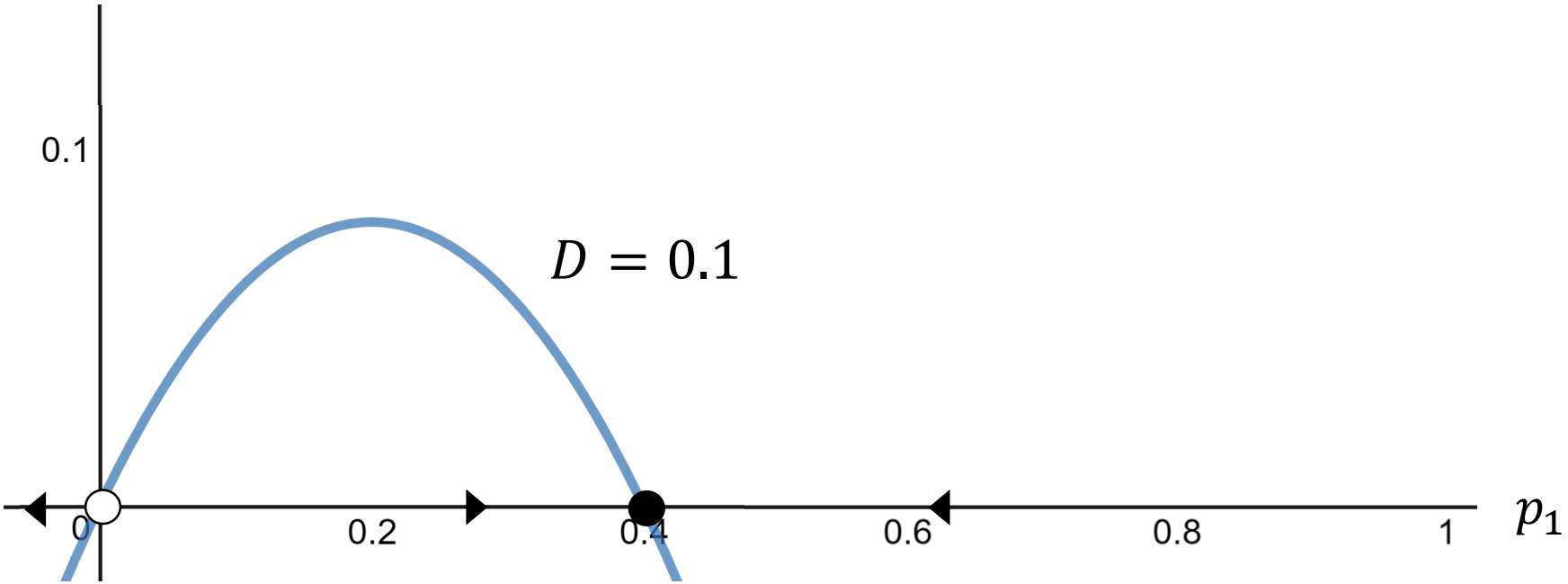


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illustration:
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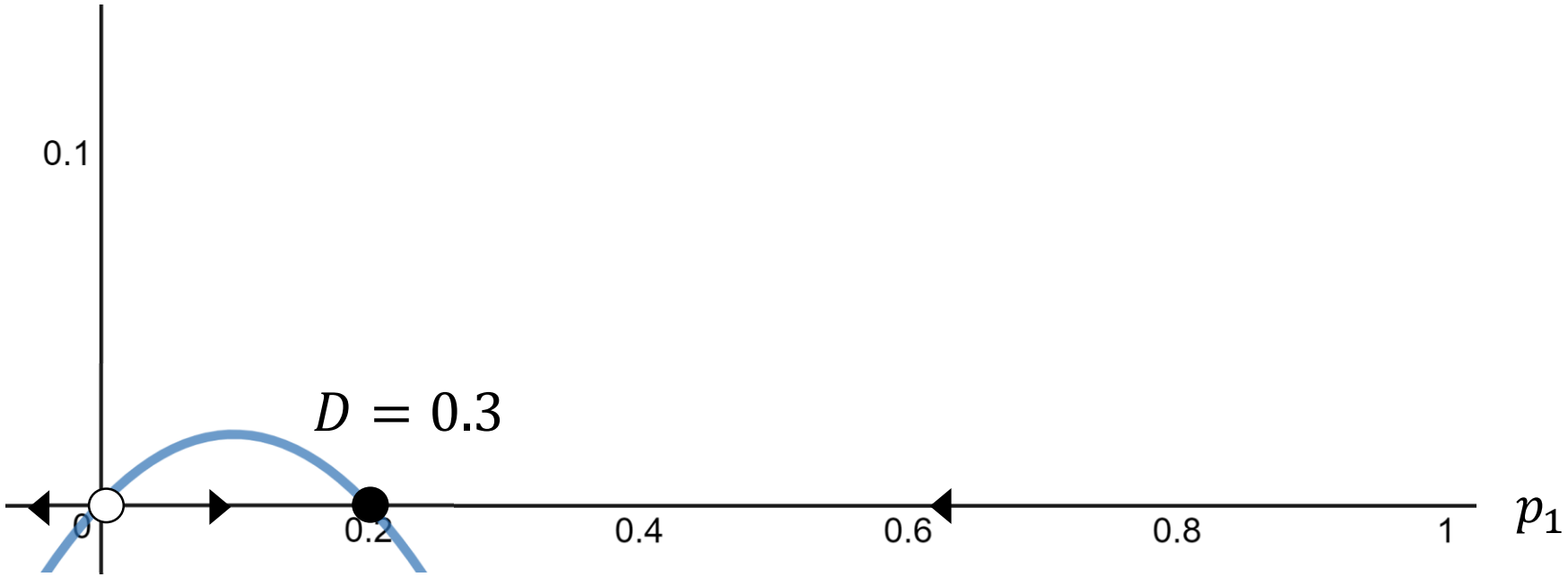


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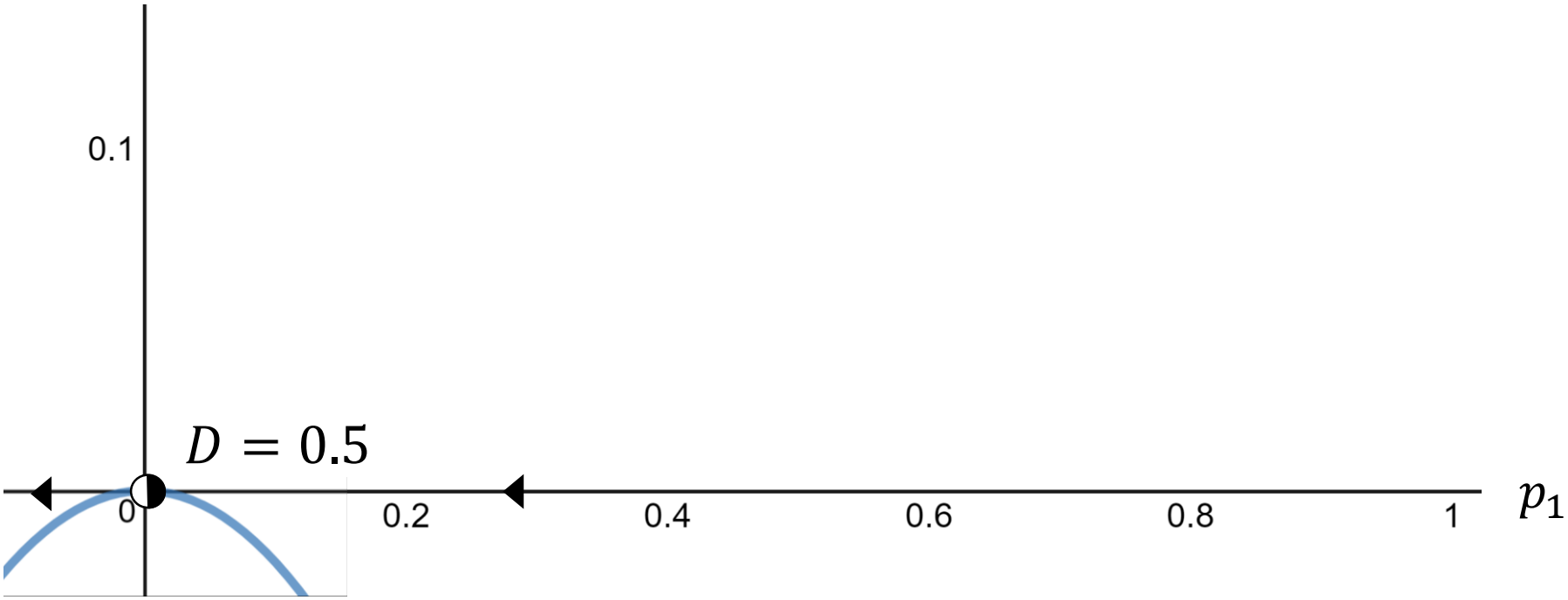


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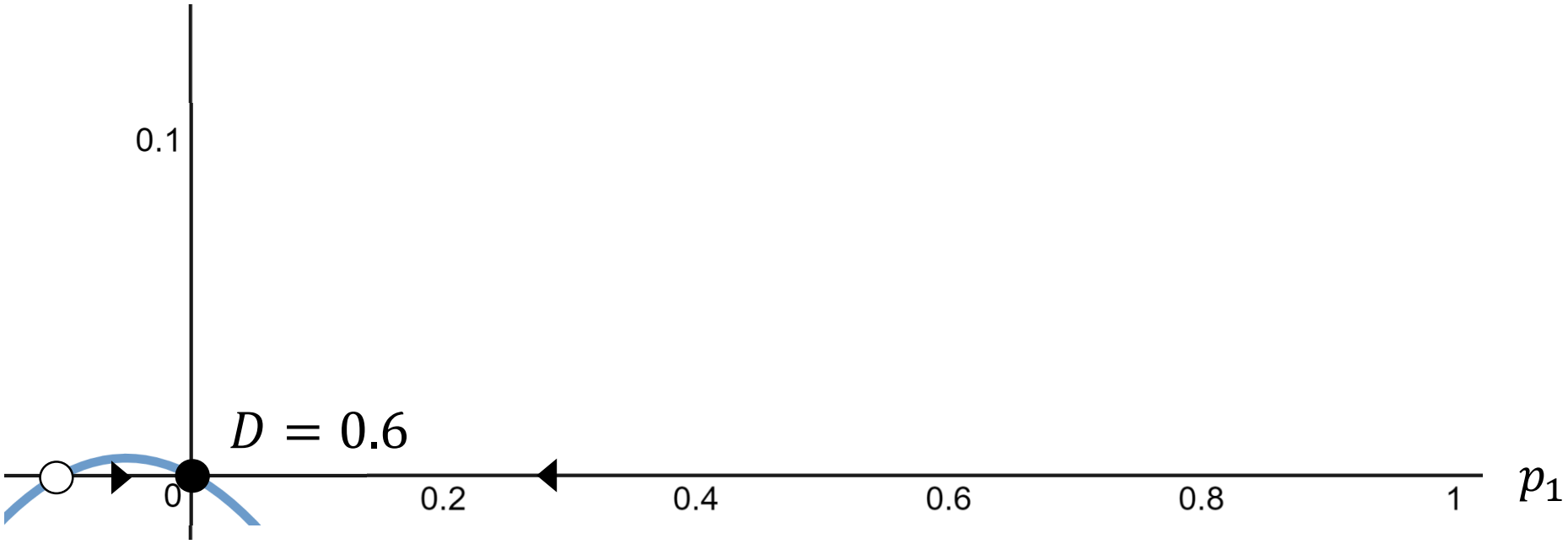
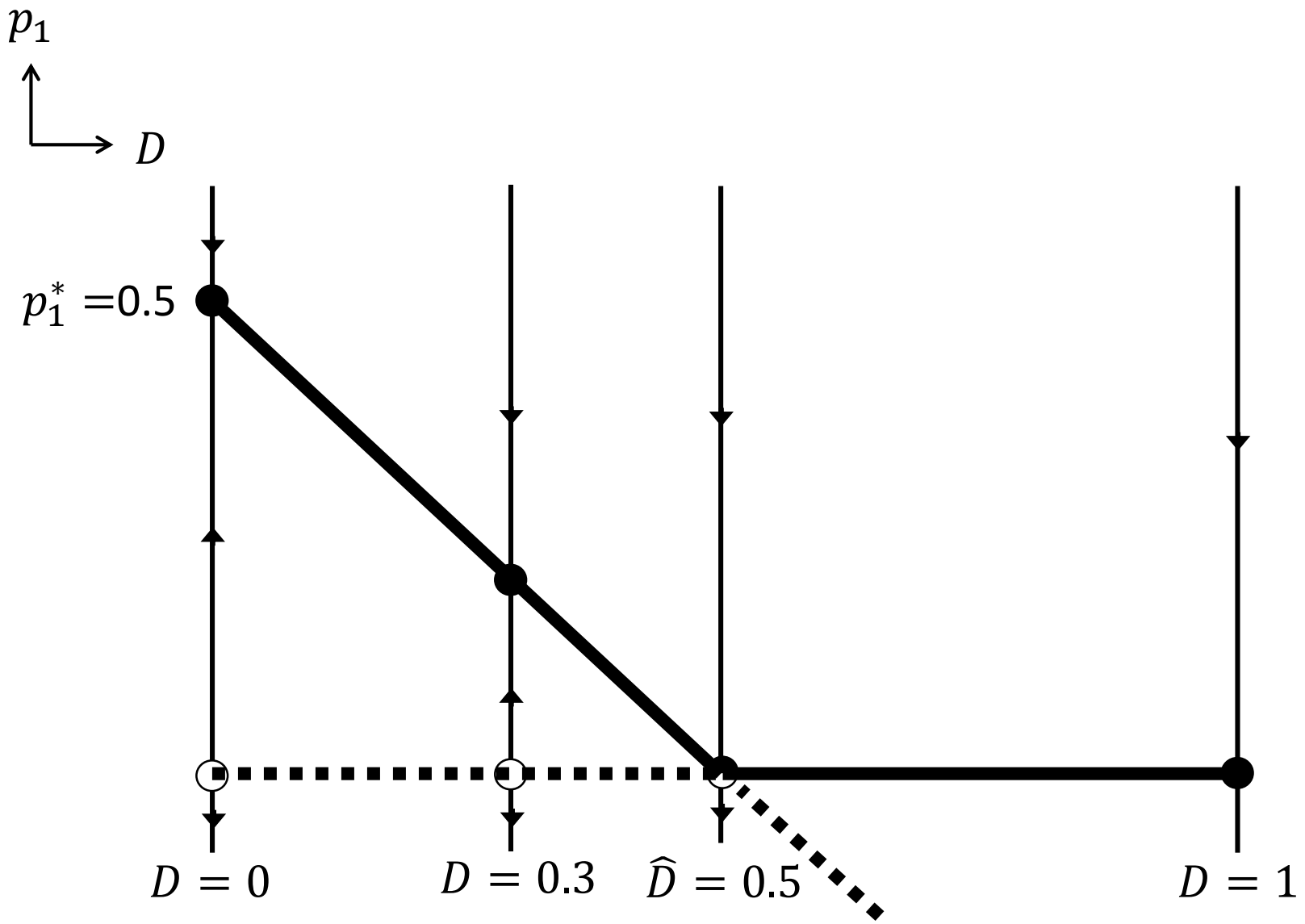
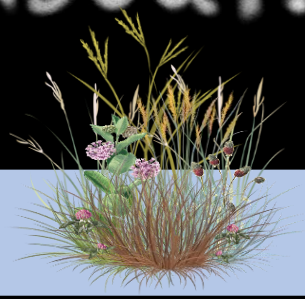



**Equilibria:** $\frac{dp}{dt} = 0 \quad \Rightarrow \quad p^* = 0, \;\; p^* = 1 - \frac{m}{c} - D$

# Population lost via transcritical bifurcation



# What can math reveal about...



**WHY** biodiversity exists?

**TRADE-OFFS**; e.g. colonization versus competitive ability

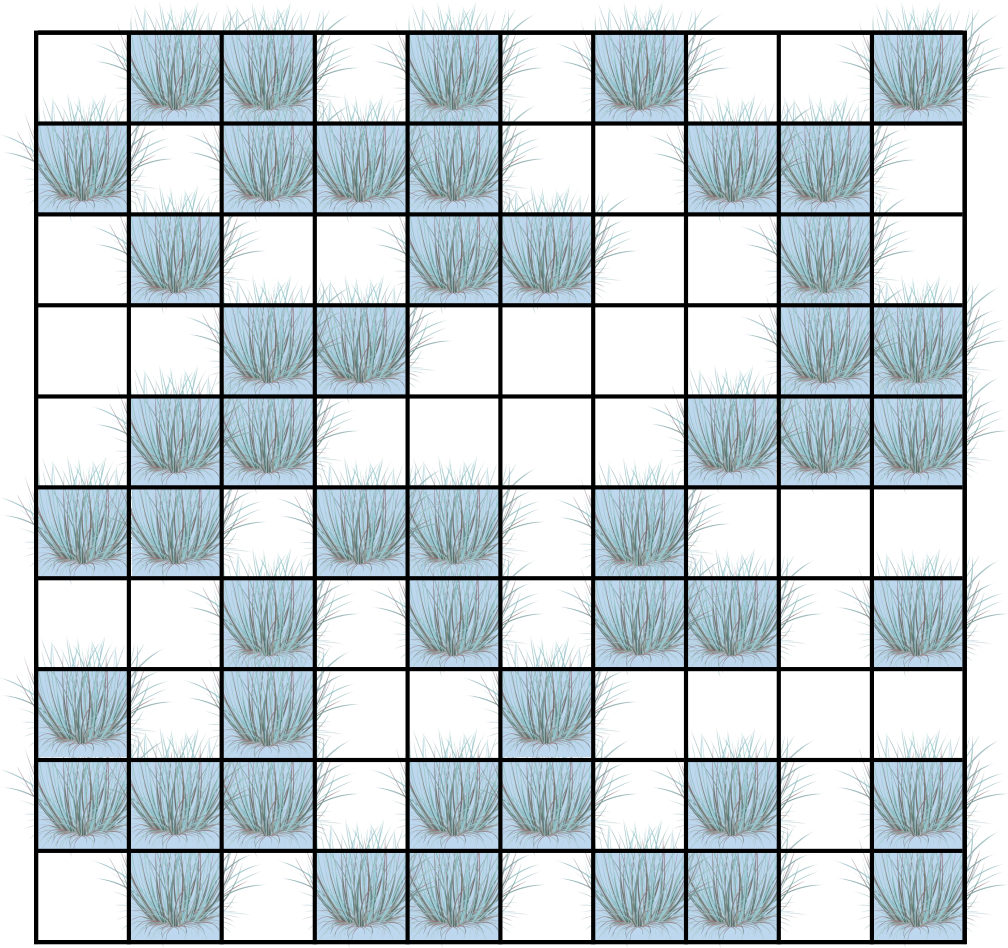
**HOW** habitat loss drives extinctions?

How much is too much?

Area equal to equilibrium abundance

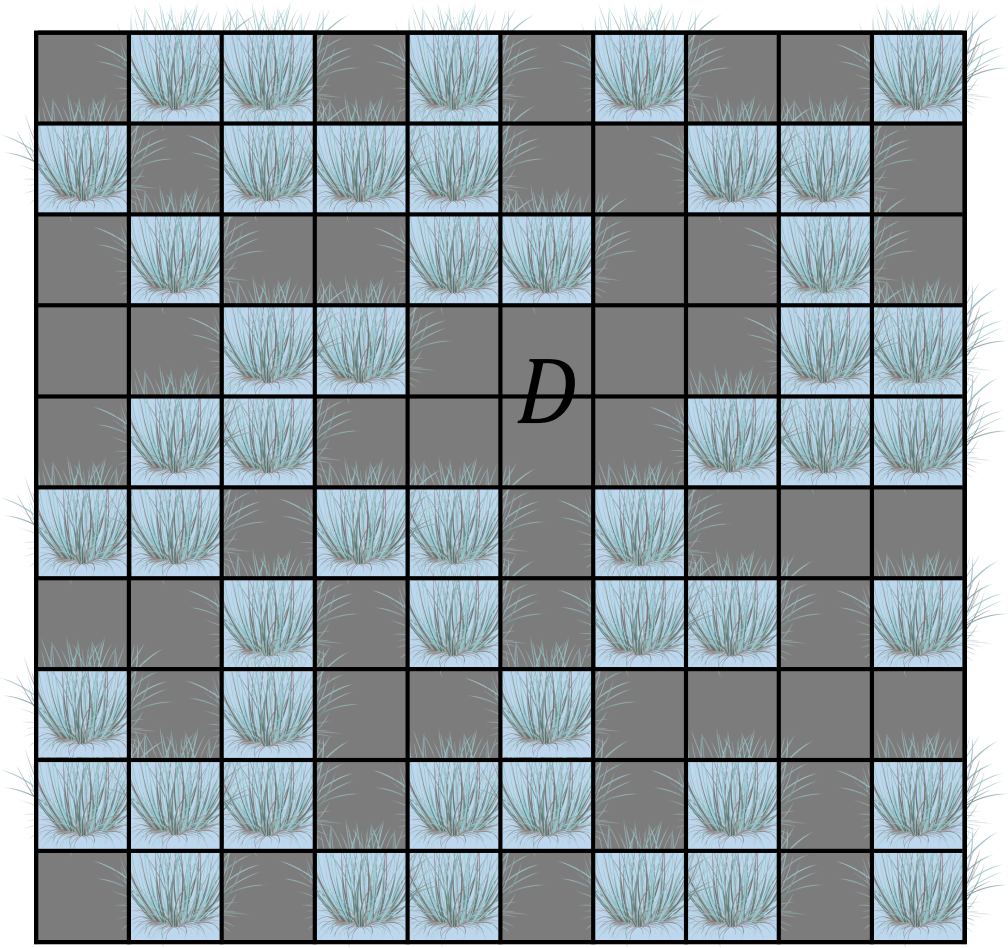
# Location of destruction is immaterial

$$D = 0 \qquad p_1^* = 0.5$$



# Location of destruction is immaterial

$$D = 0.5$$



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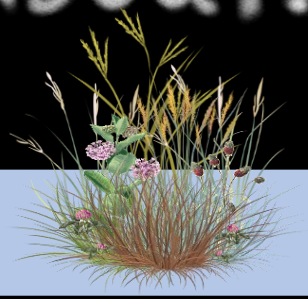


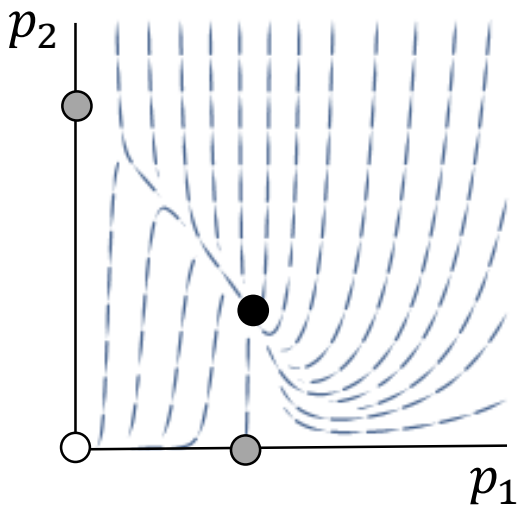
**WHY** biodiversity exists?

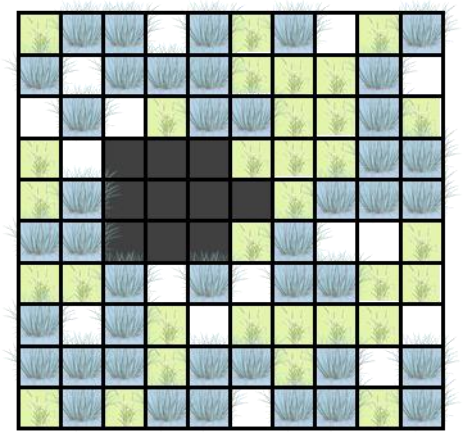
**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

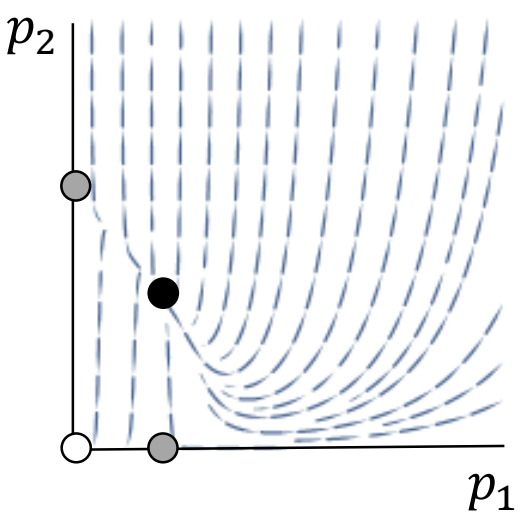
Who is at
highest risk?

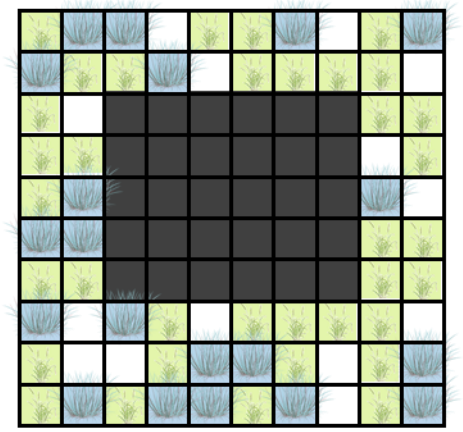
$D = 0.1$



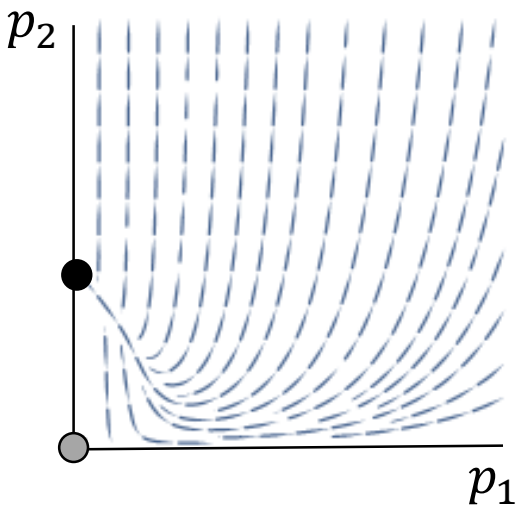



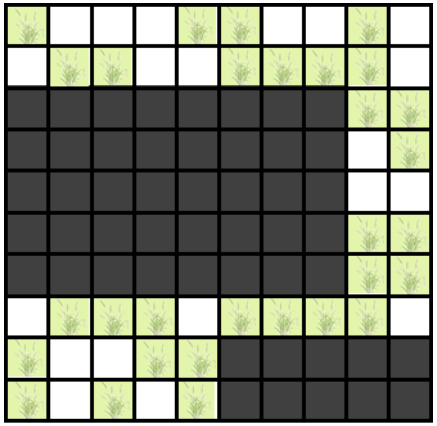
$D = 0.3$



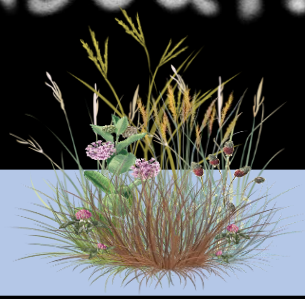



$D = 0.5$



# What can math reveal about...



**WHY** biodiversity exists?

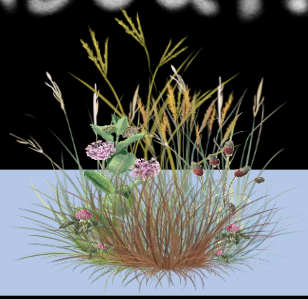
**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

Who is at highest risk?

Superior competitors

# What can math reveal about...



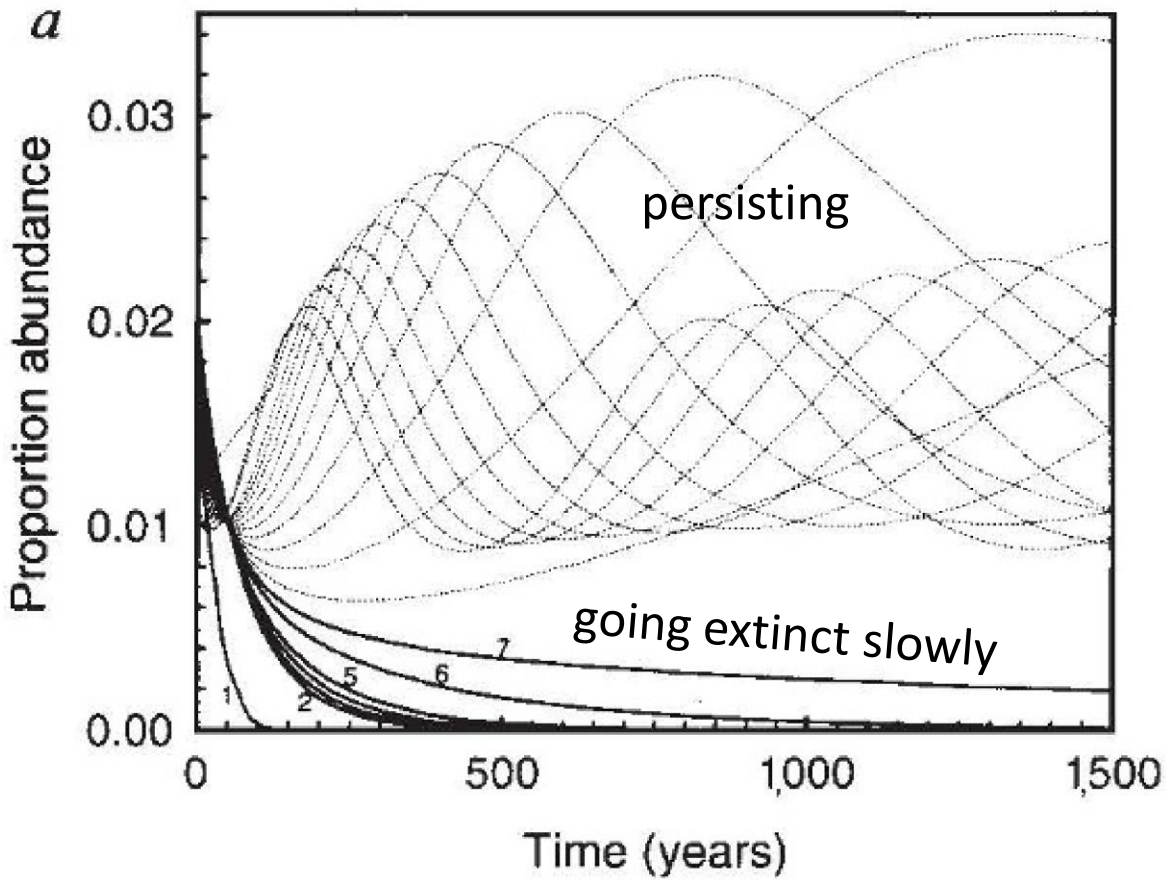
**WHY** biodiversity exists?

**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

How long
does it take?

# Extinction debts

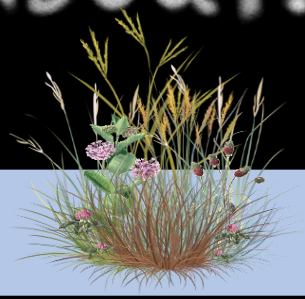



**TAKEAWAY:**

Species that are *present* after habitat loss may be slowly going *extinct*

Tilman et al. (1994)

# What can math reveal about...



**WHY** biodiversity exists?

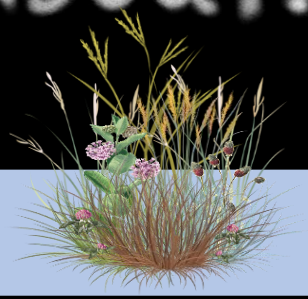
**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

How long does it take?

Possibly many generations

# What can math reveal about...



**WHY** biodiversity exists?

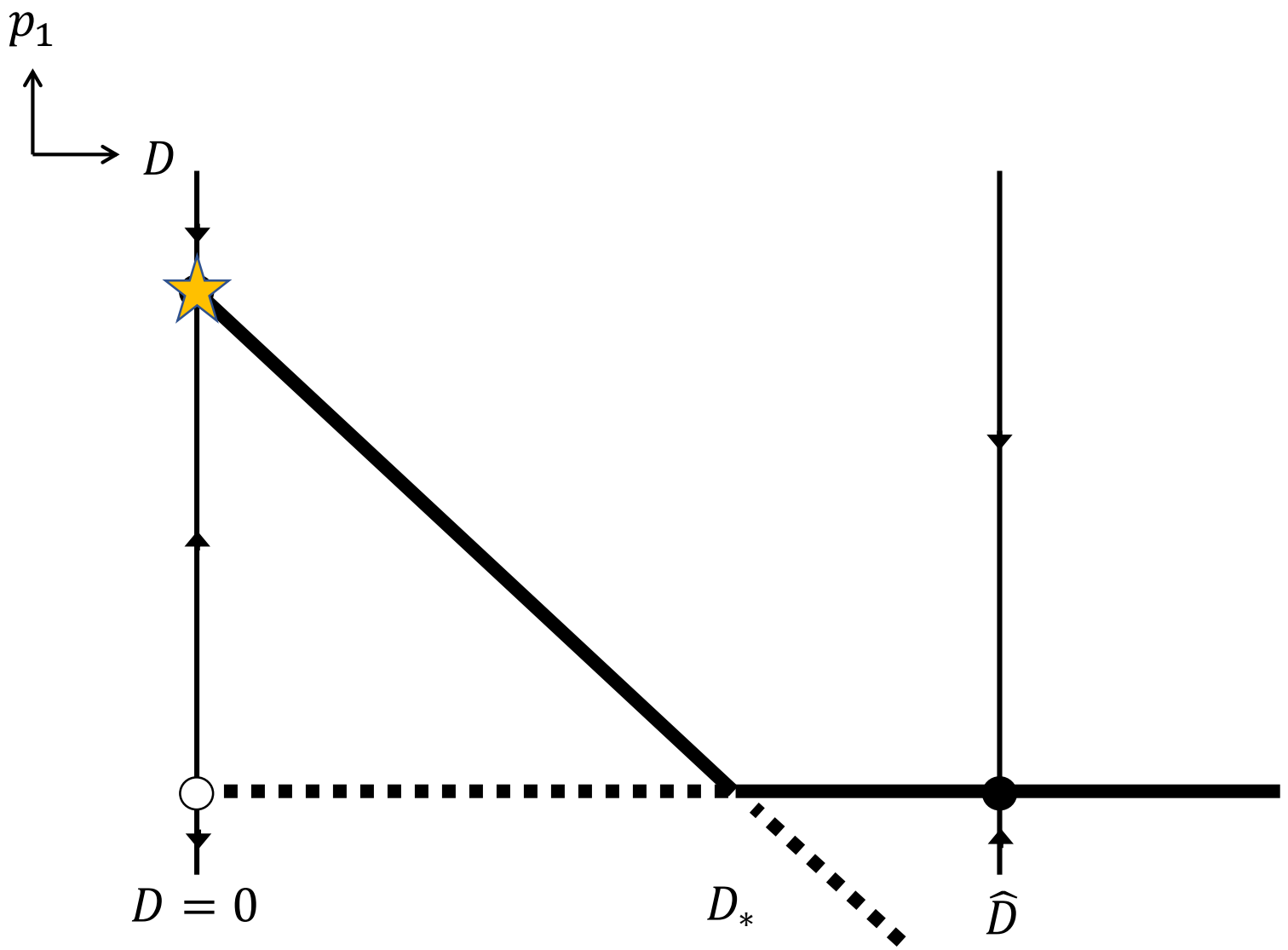
**TRADE-OFFS**; e.g. colonization versus competitive ability

**HOW** habitat loss drives extinctions?

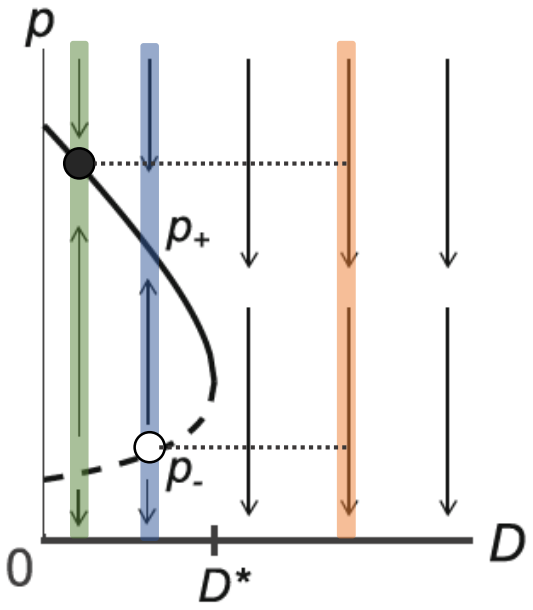
nonintuitive impacts and timing

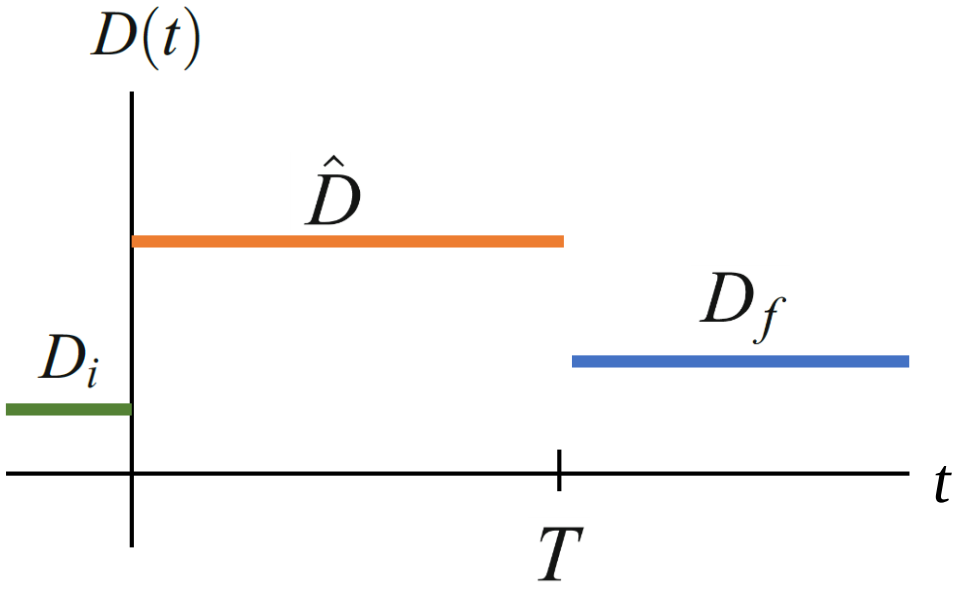
**WHAT** can be done about it?

# How long until extinction?



# Extinction debt repayment



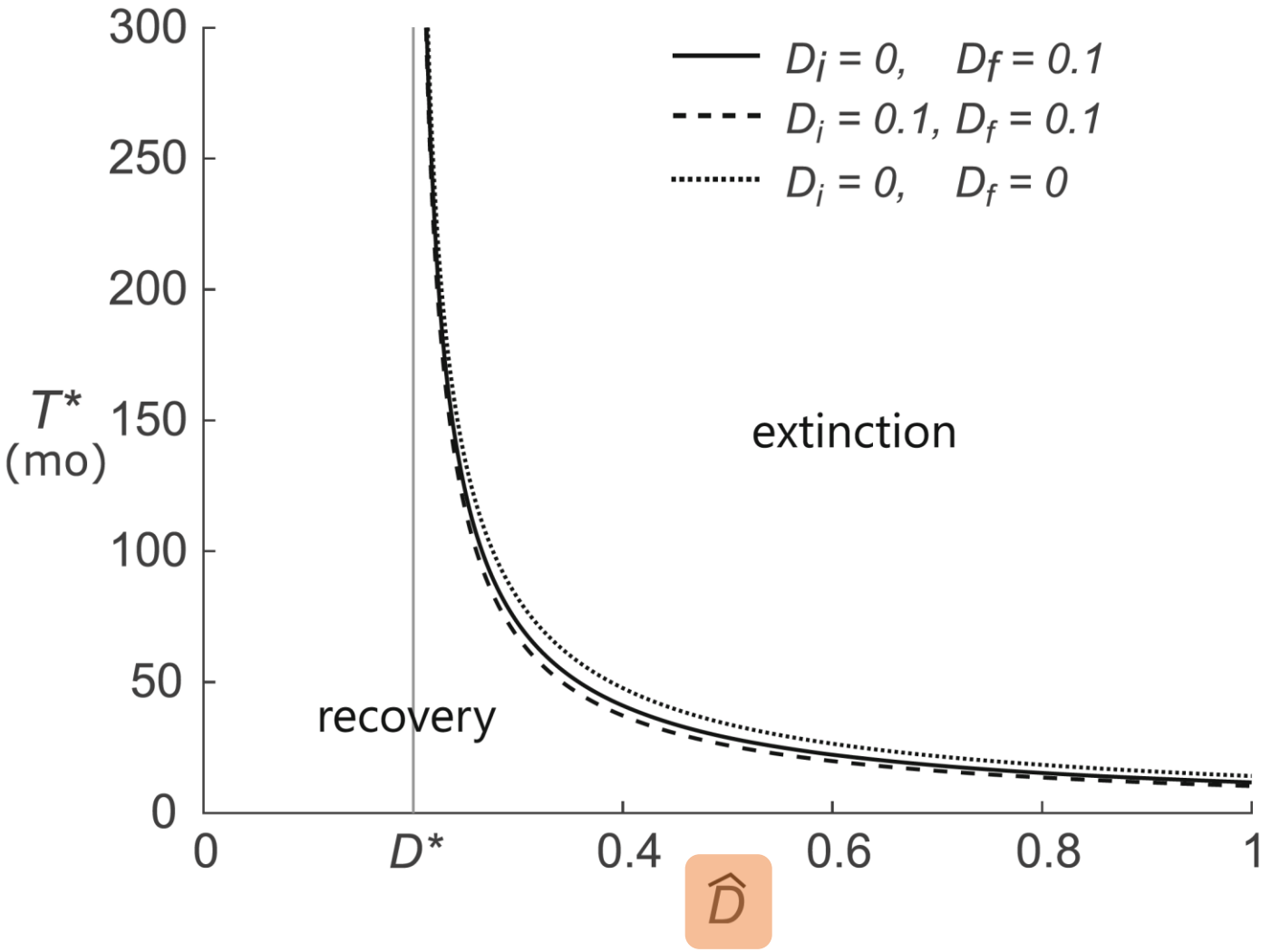

$$\frac{dp}{dt} = f\left(p, \hat{D}\right)$$

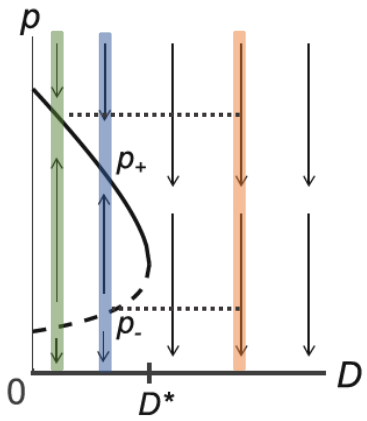
**separation of variables**

$$\int_{p_i(D_i)}^{p_{thr}(D_f)} \frac{dp}{f\left(p, \hat{D}\right)} = \int_0^{T^*} dt = T^*$$

KM (2018) Extinction debt repayment via timely habitat restoration. *Theoretical Ecology*

# Habitat restoration deadlines

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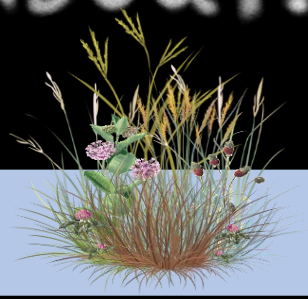


**WHY** biodiversity exists?

**TRADE-OFFS**; e.g. colonization versus competitive ability

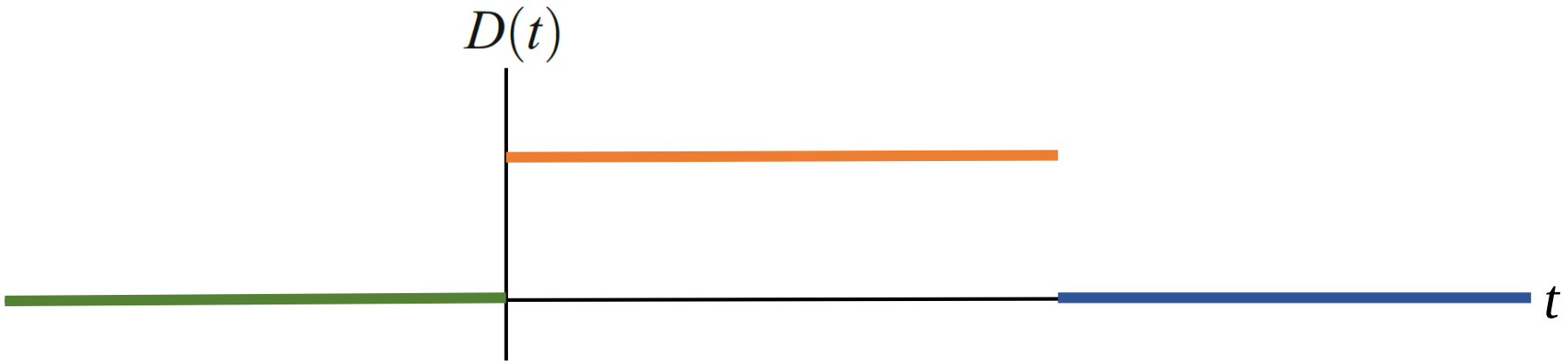
**HOW** habitat loss drives extinctions?

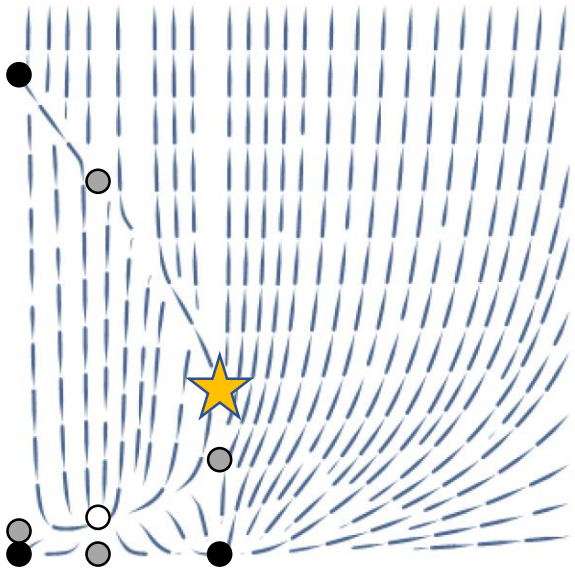
nonintuitive impacts and timing

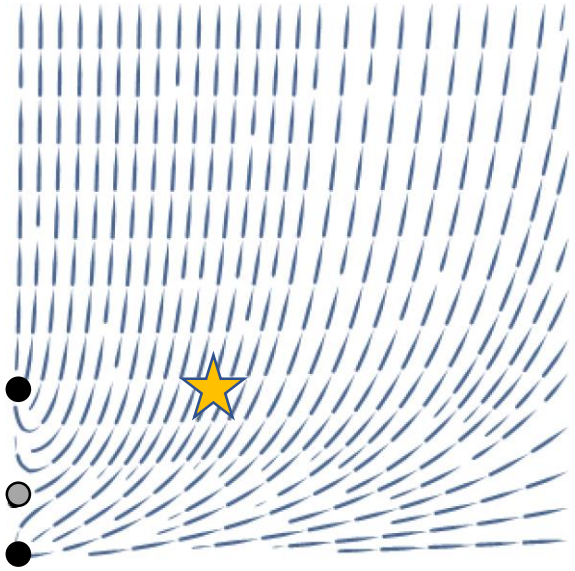
**WHAT** can be done about it?

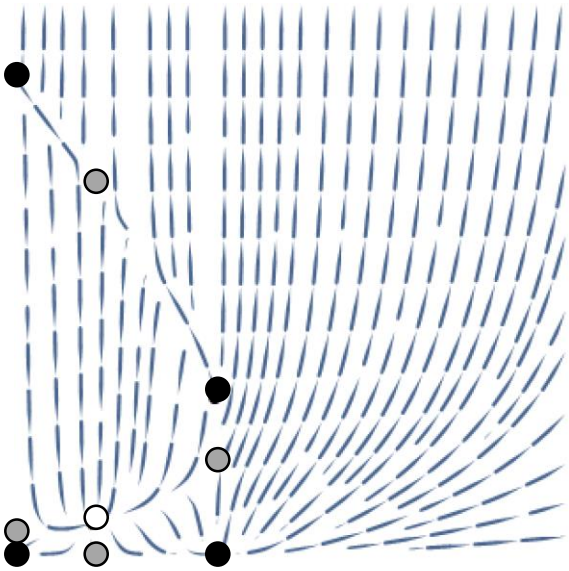
protect habitat…and restore it quickly!

# Restoration deadlines in 2D+

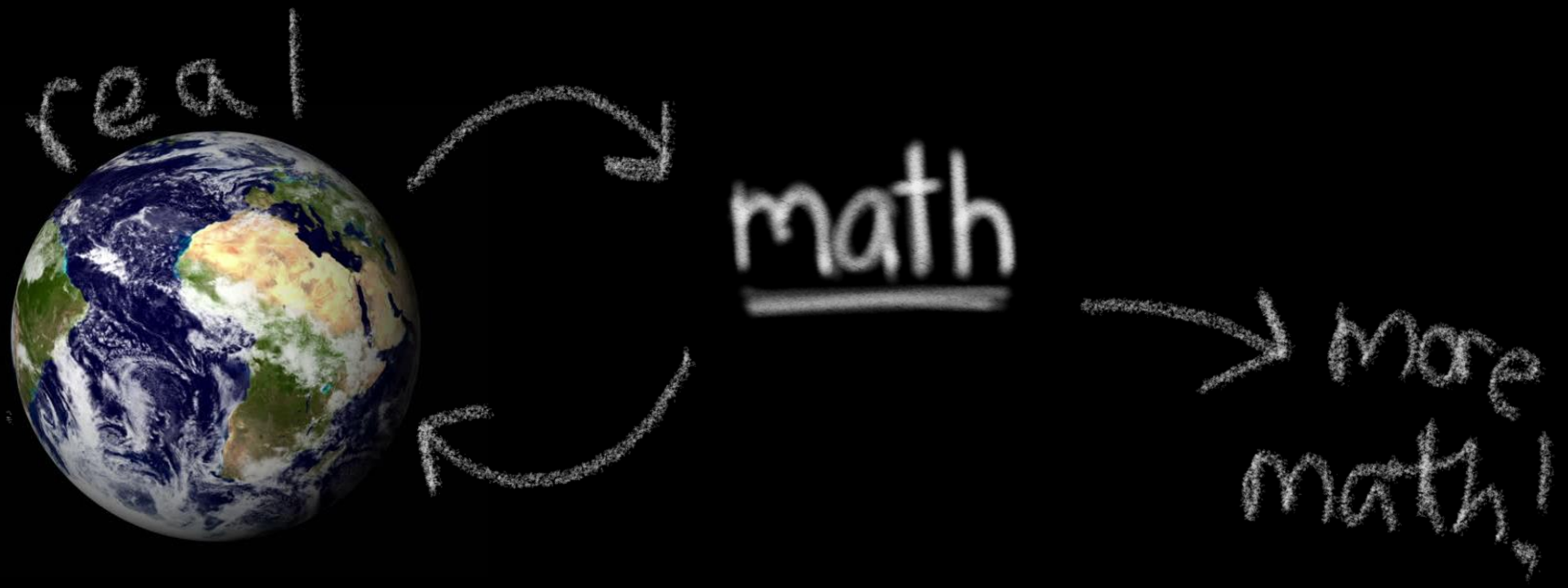






# Conclusions



# Further Reading


Cardinale et al. (2012) Biodiversity loss and its impact on humanity. *Nature* **486**:59-67.

Meyer (2018) Extinction debt repayment via timely habitat restoration. *Theoretical Ecology,* doi 10.1007/s12080-018-0395-y

Levins and Culver (1971) Regional coexistence of species and competition between rare species. *Proc Natl Acad Sci* **68**:1246-1248.

Tilman (1994) Competition and biodiversity in spatially structured habitats. *Ecology* **75**:2-16.

Tilman, May, Lehman, and Nowak (1994) Habitat destruction and the extinction debt. *Letters to Nature* **371**:65-66.

# Conditions for stable coexistence

**equilibrium**

$$\begin{bmatrix} p_1^* = 1 - \frac{m_1}{c_1} \\ p_2^* = 1 - \frac{m_2}{c_2} - \left(1 + \frac{c_1}{c_2}\right)\left(1 - \frac{m_1}{c_1}\right) \end{bmatrix}$$

**positive equilibrium** $\Leftrightarrow$

$$\begin{cases} p_1^* > 0 & \Leftarrow \quad 0 < m_1 < c_1 \\ p_2^* > 0 & \Leftarrow \quad m_1 = m_2 = m, \quad c_2 \gg c_1 \end{cases}$$

**local stability**

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} c_1 - m - 2c_1 p_1^* & 0 \\ -c_2 p_2^* - c_1 p_2^* & c_2(1 - p_1^*) - 2c_2 p_2^* \\ & -m - c_1 p_1^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$< 0$ $< 0$